Tax Evasion, Corruption and Tax Loopholes

Sugata Marjit
André Seidel
&
Marcel Thum

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Sugata Marjit*, André Seidel** & Marcel Thum***

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Abstract:

This paper addresses tax loopholes that allow firms to exploit borderline cases between legal tax avoidance and illegal tax evasion. In general, tax loopholes are detrimental for a revenue-maximizing government. This may change in the presence of corruption in the tax administration. Tax loopholes may serve as a separating mechanism that helps governments maximize revenues and curb corruption, which may explain why developing countries only gradually close loopholes in their tax codes.

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* Centre for Studies in Social Sciences, Calcutta, R1, Baishnabhata Patuli Township, Kolkata, India.
** Faculty of Business and Economics, TU Dresden, 01062 Dresden, Germany, andre.seidel1@tu-dresden.de (corresponding author).
*** Faculty of Business and Economics, TU Dresden, 01062 Dresden, Germany; ifo Dresden and CESifo, marcel.thum@tu-dresden.de. Financial support from the Deutsche Forschungsgemeinschaft (Grant 759-3) is gratefully acknowledged.
1. **INTRODUCTION**

In many developing countries, governments face a serious dilemma when raising tax revenues. Fiscal resources are urgently needed to finance essential state services, invest in infrastructure and provide the necessary inputs for future growth. These fiscal resources are often supposed to come from a fairly small number of financially sound firms and wealthy individuals, rather than from the relatively poor masses. However, these firms frequently find ways to escape their fiscal burdens using elaborate strategies to avoid and evade taxes.\(^1\) The government may counter such tax evasion and avoidance by hiring additional tax officials to monitor firms more closely. If these tax officials are corrupt (with low tax revenues, there is little capacity to provide sufficient financial incentives for honest behavior), the government’s efforts may be frustrated because the tax official may become a partner in crime in exchange for a bribe. This simplistic initial view on taxation in the presence of corruption suggests that governments may be trapped in settings with low tax revenues and high levels of corruption.\(^2\) Nonetheless, surprisingly little scholarly research examines the combination of tax evasion, tax avoidance and corruption in tax administrations. Most studies so far have addressed corruption and tax evasion for households [see for example Chander & Wilde (1992), Besley & McLaren (1993), Hindricks, Keen & Muthoo (1999), Marjit, Rajeev, & Mukherjee (2000), Cerqueti & Coppier (2009)]. This body of literature is closely related to the economics of crime, and it thus focuses primarily on containing crime with appropriate incentive schemes (wages, fines, etc.). Obviously, there is a comprehensive body of literature that addresses tax evasion in general [see Slemrod (2007) for a survey]. In the early literature on tax evasion by firms, the standard model of household tax evasion was simply transferred to firms; for a survey of this literature, see, for example, Cowell (2004). The more recent literature uses new types of models to explore related matters, such as the linkage between competition and tax evasion [Goerke & Runkel (2006; 2011)] or the internal costs of control of tax evasion to a firm [Chen & Chu (2005), Crocker & Slemrod (2005)]. Tax avoidance plays no role in these studies.

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\(^1\) The same argument also applies to self-employed persons. The crucial point is that the opportunities for tax evasion increase with the share of self-reported income [Kleven (2014)]. Employees are typically taxed via third-party reporting and have little leeway for tax evasion.

\(^2\) Besley & Persson (2010) discuss the efforts to escape this trap as capacity building.
We are interested in the borderline cases between tax avoidance and tax evasion. Due to
the complexities of modern day businesses, it is not sufficient to lay down the
fundamental rules in the tax code. In addition to the tax law, many detailed
implementation rules must clearly state the government’s claim for taxation. Otherwise,
clever tax accountants will manipulate the tax base, such as by claiming that a firm
owner’s private expenditures are business-related costs. The number of pages of
primary federal tax legislation demonstrates the scale of implementation rules. In the
United States, Japan and the United Kingdom, federal tax legislation covers 5,100, 7,200
and 8,300 pages, respectively; however, in Turkey, Russia and Mexico, that same
legislation covers only 350, 700 and 1600 pages, respectively.\(^3\) The lack of detailed
implementation rules may lead to a type of legal limbo between tax evasion and
avoidance, which is characterized by the prevalence of “tax loopholes”.

Tax authorities around the world have large difficulties in drawing a clear line between
tax evasion and avoidance. “Commissioners frequently complain of the lack of good
managerial information for timely decisions” (IMF, 2015, p.40). The recent discussion in
the public media on so-called cum/ex trades in which transactions around dividend,
payments were used for tax arbitrage, are a good example for this. Developed and
developing country struggle at the moment to close this tax loopholes. For example,
India and Swaziland only recently introduced laws that aim at preventing tax avoidance
related to cum/ex trades.\(^4\) The complexity of the tax arbitrage makes it difficult even for
highly developed economies with comprehensive tax codes to close the tax loopholes.

For instance, Bloomberg reported on investigations against former employees of
Deutsche Bank. One accused banker claimed that “[t]hese transactions were legal when
they took place as all tax experts said at the time,”\(^5\) but German tax authorities thought
differently and alleged that “firms fraudulently obtained hundreds of millions of dollars’

\(^3\) Source: Paying Taxes: The Global Picture published by PricewaterhouseCoopers and The World Bank, 2006. The relatively small quantity of implementation rules might also reflect a lower demand for such rules in developing countries. However, the International Monetary Fund puts the extension of the legal framework high on the agenda: “Much remains to be done in many countries to build effective tax administrations ... recurrent concerns, ... —especially in developing countries—are weaknesses in auditing, taxpayer services and legal frameworks.” (IMF, 2015, p. 2).


worth of tax benefits." In the same article, the Wall Street Journal Europe explicitly referred to the tedious job of closing these tax loopholes: “The market for cum/ex trades largely died off in 2011, when tax authorities closed loopholes and exchange officials fine-tuned how they handled certain transactions [...]”

In less developed countries, tax disputes are often not well documented but there is some anecdotal evidence that tax loopholes play a significant role, as the example of India and Swaziland has shown. Some of these tax loopholes may even be kept on purpose. For example in South Africa, taxpayer can ask the tax authority for a binding ruling with respect to any transaction [Dalton 2012]. As the tax authority is bound by its rulings, this procedure reduces the risk of wrongdoing on the taxpayer’s side. Taxpayers, however, are not forced to follow tax authority’s rulings and can treat a transaction contrary to the ruling. As the rulings are not binding for the tax payers, they do not eliminate tax loopholes but rather give clever tax arbitragers some guidance how to exploit tax loopholes. Another sign for the lack of interest in closing tax loopholes may is the hesitation of many developing countries to participate in the Base Erosion and Profit Shifting (BEPS) project of the OECD. BEPS aims at reducing international tax competition but also at closing tax loopholes that are frequently exploited by multinational corporations. Only 16 developing countries actively participate in the BEPS negotiations. Some countries may simply have no interest in pushing the case for international tax coordination as they benefit from being low tax jurisdictions. Overall, however, this is a small number of countries. Most low- and middle-income countries could benefit from an internationally coordinated and clearly defined tax base as they typically do not qualify as tax havens, e.g., due to high levels of corruption (Dharmapala & Hines, 2009). One example is “Preventing the Artificial Avoidance of Permanent Establishment Status” (Action 7) of the BEPS. This action plan tries to improve the definition of establishments. This is particularly relevant for the digital economy where firms can easily generate profits in a country but at the same time avoid taxation in this country as long as the BEPS rules are not adopted. Hence, low- and middle-income

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7 Another 61 developing countries were involved in regional network meetings, which were mainly designed for information exchange, but they did not actively participate in the BEPS process; see http://www.oecd.org/tax/developing-countries-and-beps.htm.
countries could improve their tax base by re-defining the establishment status according to the BEPS rules but are still hesitant to do so.

There are various explanations for the particularly fuzzy implementation rules in developing countries. First, some countries may simply lack fiscal capacity. Without profound expertise in the finance ministry and in the tax administration, it is impossible to fix detailed rules for complex accounting transactions. The government has to leave decisions on complex tax issues to the discretion of the tax officials. Second, the tax administration itself may have an interest in an incomplete tax code. This gives the administration some discretion in setting the firms’ tax liabilities. Bureaucrats in the tax administration may lobby to prevent the elimination of tax loopholes. In a third explanation, the government itself may strategically create tax loopholes to fight tax evasion via corrupt tax officials. Corrupt tax officials help firms hide income from the government by rubberstamping manipulated tax files. Firms can either bribe corrupt officials or try to exploit the loopholes. In the former case, they will have to pay the bribe and therefore share the tax savings with the corrupt tax official. In the latter case, they may be punished in court but keep the full tax savings when successful. Which strategy is more profitable for a particular firm, will depend on the expertise of the firm’s accountants and the size of the tax loopholes. If the government leaves a sufficient number of tax issues undecided, it makes it more attractive for firms with able tax accountants to focus on tax loopholes rather than on colluding with corrupt tax officials. Thus, increasing the volume of tax loopholes can reduce the total revenue loss from tax evasion and curb corruption. We do not want to claim that this third mechanism is empirically more important than the other channels. We just want to highlight that, in the presence of corruption, governments may have an incentive to close not all loopholes. This mechanism might seem paradoxical at first sight and – to our knowledge – has not been analyzed in the literature so far.

We establish a simple model that allows us to analyze the role of tax loopholes in an economy in which officials in a corrupt tax administration support tax evasion by firms. In the benchmark scenario without corruption (Section 2), firms face a tradeoff between the benefits from exploiting tax loopholes and the risk of being found guilty of tax evasion. We assume that firms differ in their likelihood of being sentenced. Firms that for example have more experienced accountants and top legal experts are less likely to
be punished in court because they will be able to use tax loopholes and create borderline cases between illegal tax evasion and legal tax avoidance. In the second scenario (Section 3), the government employs corrupt tax officials who perfectly monitor each firm. The firms are informed about their individual abilities to exploit tax loopholes regarding a lawsuit, and the corrupt officials know only the distribution of abilities. The firm and the corrupt official may become partners in crime, in which case the corrupt official receives a bribe in exchange for rubberstamping the firm’s tax files. Firms with more able accountants, however, may find it profitable to ignore the official’s bribe demand and risk a verdict of tax evasion in court. Whether the corrupt official makes an offer that is attractive for less and more able firms will depend on the size of the tax loopholes. The larger the scope of the tax loopholes, the more attractive the loophole option is for more able firms. The government can exploit this mechanism precisely at this juncture. By maintaining a sufficient volume of tax loopholes, it can entice more able firms not to bribe tax officials. Tax loopholes limit the power of corrupt officials and, therefore, change the bribe demanded from the firms. In Section 4, we show that the result of our admittedly simple model also holds when only a fraction of tax officials are corrupt. Section 5 analyzes the case in which the protection offered by corrupt officials is incomplete. Firms that pay the bribe still have a positive probability of being sentenced for tax evasion, and we show that the corrupt official may have an incentive to target more able rather than less able firms in this case. However, the main result that the government may not want to close all tax loopholes still holds. In Section 6, we show that the strategic role of tax loopholes still prevails when we extend the strategy space of the government and allow for purely random tax audits, which are carried out independent of the tax official’s assessment of individual tax files. Section 7 concludes.

2. A SIMPLE MODEL OF TAX LOOPHOLES

We first establish a model for tax loopholes before we introduce corruption. To keep things simple and to maintain consistent wording, we will use the example of tax loopholes for a tax on corporate profits. We assume that the precise tax liability of a firm is, to some extent, a matter of dispute. The tax code may define taxable profits, but calculating the tax base will always allow some room for maneuvering, which typifies borderline cases between tax avoidance and tax evasion. If a firm wants to be entirely
safe, it will not engage in accounting transactions that might potentially be classified as tax evasion by the tax authorities. Alternatively, a firm can reduce its tax liability by amount $e$, which might be judged as tax evasion in a lawsuit. All the tax statements of firms are inspected by a tax official. If the tax official learns about the disputable tax liabilities, he or she will forward the case to superiors who carry out a detailed tax audit and initiate a lawsuit when $e > 0$. If the court classifies the firm’s tax planning as tax evasion, the firm must pay the tax. In addition to the payment of the back taxes, the entrepreneur also suffers the moral cost of being characterized as a tax evader; the monetary equivalent of this moral cost is denoted by $c$.

Firms differ with respect to their ability to use tax loopholes. Firms with more experienced accountants and legal experts are less likely to be punished in court because they can create borderline cases between illegal tax evasion and legal tax avoidance. Furthermore, there are ambiguities in the tax code that can be exploited more easily by certain firms. For example, the borderline between taxable and tax exempt entities is often fuzzy. For purposes of simplicity, we assume that there are two types of firms: $i = H, L$. The probability that the accounting transactions of type $H$ firms will be judged as tax evasion is $g < 1$. The probability that the transactions will be classified as legal tax avoidance is $(1 - g)$. The parameter $g$ can also be understood as a proxy for the completeness of the tax code. A high $g$ indicates an almost complete tax code that makes it difficult for firms to exploit tax loopholes without losing a lawsuit. We will refer to an increase in $g$ as the closing of tax loopholes. Firms of type $L$ will always lose such lawsuits and will suffer the moral cost of tax evasion. Therefore, firms of type $H$ face a lower expected cost for the same amount of potential tax evasion. A fraction $\beta$ of all firms are type $L$, and the share of $H$-type firms is $1 - \beta$.

The expected payoff from exploiting the tax loopholes amounts to the following:

$$\pi_H = (1 - g) \cdot e - g \cdot c$$  \hspace{1cm} [1]

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8 For simplicity, we will refer to $e$ as tax evasion, although some firms may be acquitted.

9 Alternatively, we might assume a penalty for the attempted tax evasion, which would, however, raise the question whether the penalties should be included as government revenues. The assumption of a moral cost avoids such a revenue effect. The main results of the paper do not depend on the specific modeling approach.

10 The results would not change if firms of L-H-type had a positive chance of succeeding in a lawsuit but with a probability lower than $g$. An alternative way of modeling heterogeneity is to assume that the firms of $H$-type will be able to lower the evasion charges in a lawsuit by more than the L-type firms.
\[ \pi_L = -c \]  

for types H and L, respectively. Hence, the L-type firms would never risk using the tax loopholes, and the H-type firms will exploit tax loopholes if the risk of conviction is sufficiently low \((\pi_H > 0 \Rightarrow g < \frac{\epsilon}{\epsilon + c})\). For a revenue-maximizing government, closing tax loopholes \((g = 1)\) is always beneficial in a world without corruption.

3. **TAX LOOPHOLES AND CORRUPTION**

We now introduce corruption into the tax administration. Thus, the tax officials who are supposed to monitor firms are willing to support a firm in its potential tax evasion \(e\) in exchange for a bribe \(b\).\(^{11}\) To keep things simple, we assume that the tax official can reduce the probability of a trial to zero, such as by rubber-stamping a tax file.\(^{12}\) If a firm refuses to pay a bribe, it can nonetheless choose a positive amount of potential tax evasion \(e\), but the official will report the case to his superiors and the firm will face a trial with certainty. We analyze a two-stage game in which the (representative) corrupt official sets the bribe rate before the firms submit their tax returns.

Firms have three options: (i) exploit the tax loopholes and pay a bribe, which yields \(e - b\); (ii) exploit the loopholes without bribing the officials, which leads to the expected payoffs as stated in [1] and [2]; or (iii) engage in no disputable accounting transactions, which leads to the payoff normalized to zero. The L-type firms will either stay on the safe side, not risking any potential tax evasion, or pay the bribe to exploit the tax loopholes with the support of the corrupt official. The potential tax evasion will be profitable if \(b \leq e\). For H-type firms, the outside option depends on the risk \(g\) of being convicted as a tax evader. For high H-risk \(g\), the trade-off is the same as that for the L-type firms. For \(g < \frac{\epsilon}{\epsilon + c}\), however, the firms would risk the evasion even without support from the corrupt official. The H-type firms will accept the corrupt official’s offer only if \(b \leq g \cdot (e + c)\).

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\(^{11}\) This is equivalent to a bargaining situation in which the corrupt tax official makes a take-it-or-leave-it offer. We might also allow for a fraction of the tax officials to be honest, and honest tax officials report all suspected instances of tax evasion. We focus on the extreme case of comprehensive corruption to illustrate our point. For an extension of the model in this direction, see Section 4.

\(^{12}\) In Section 5, we will analyze a variant of the model in which the probability of detection remains positive even when a bribe has been paid.
The corrupt official knows the distribution of types but cannot identify the type of a single firm. This assumption does not imply that the corrupt official cannot discriminate at all. Clearly, bribes may be contingent on some observable characteristics such as size, revenue or number of employees of a firm. However, firms with identical observable characteristics may still differ along many other, unobservable dimensions such as the ability and experience to handle complex accounting transactions. For simplicity, we do not model differences in observable characteristics but rather focus on the heterogeneity in unobservables. This situation leaves the official with two alternatives: he or she can either charge a bribe that is acceptable for both types of firms (pooling) or target the L-type firms only (separating). Targeting H-type firms only is not possible; any offer that H-type firms accept will always be accepted by L-type firms as well. With separating, the corrupt official charges $b_{SEP} = e$, making the L-type firms indifferent. The total gain from corruption for the official amounts to $B_{SEP} = \beta e$. With pooling, the bribe depends on the H-type firms' outside option:

$$b_{POOL} = \begin{cases} 
  g \cdot (e + c) & \text{for } g \leq \frac{e}{e + c} \\
  e & \text{for } g > \frac{e}{e + c} 
\end{cases}$$

[3]

The total bribe received by the corrupt official amounts to $B_{POOL} = g \cdot (e + c)$ and $B_{POOL} = e$. Figure 1 summarizes the profit-maximizing strategies for corrupt officials. The probability of defeat in court for H-type firms is denoted on the horizontal axis, where the vertical axis displays the share of L-type firms. For $g \leq \frac{e}{e + c}$ the official must engage in a trade-off between the benefits from a larger ‘customer base’ in the case of pooling and the higher bribe from separating. For $\beta > g \cdot \frac{e + c}{e}$, i.e., for a high share of L-type firms or for a low probability $g$ of punishment for the H-type firms, the corrupt official will find it optimal to only target L-type firms.

...
Figure 1. Pooling and Separating

The exploitation of tax loopholes leads to revenue losses for the government. Let \( \Delta T \) denote the expected revenue loss of the government, including back taxes that must be paid from firms convicted for tax evasion. The revenue loss can be written as follows:

\[
\Delta T = \begin{cases} 
\Delta T_{SEP} & \text{for } g \leq \beta \cdot \frac{e}{e + c} \\
\Delta T_{POOL} & \text{for } g > \beta \cdot \frac{e}{e + c}
\end{cases}
\]

In the case of pooling, all firms use tax loopholes with the assistance of corrupt officials. The government loses revenue \( e \). With separating, all firms again attempt to use the tax loopholes, but only the \( L \)-type firms are assisted by the corrupt officials. The \( H \)-type firms, which have a share of \( 1 - \beta \), are convicted for tax evasion with probability \( g \) and must pay \( e \). The punishment probability \( g \) also reflects the ambiguity of the tax code. If \( g = 0 \), all transactions will be classified as legal tax avoidance by the court, and the government will lose revenues due to a smaller tax base. If \( g = 1 \), the tax code is so elaborate that all efforts to exploit tax loopholes will fail; these transactions are always classified as illegal tax evasion by the courts. In a world without corruption, the government would like to set \( g = 1 \). However, corrupt officials can hide the disputable cases, and turn potential tax revenues into bribe payments. If the government wants to maximize tax revenues, it should thus not eliminate all tax loopholes.

**Proposition 1.** A government that wants to minimize the expected revenue will not eliminate all tax loopholes. The expected revenue loss for the government is minimized when \( g^* = \beta \cdot \frac{e}{e + c} \).
Proof. The revenue loss $\Delta T_{SEP}$ never exceeds $\Delta T_{POOL}$ and decreases in $g$. Therefore, $g^* = \beta \cdot \frac{e}{e+c} < 1$ minimizes the revenue loss. ■

Figure 2 shows the government’s revenue loss as a function of the detection probability $g$. For very low levels of $g$, an increase in the detection probability is beneficial for the government because more of the $H$-type firms are punished as tax evaders. However, by increasing $g$ above the critical threshold, the government induces the $H$-type firms to bribe corrupt officials. When $g^* = \beta \cdot \frac{e}{e+c}$, the $H$-type firms are indifferent between evading on their own and bribing officials.

**Figure 2. Total Tax Evasion**

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**Proposition 2:** The revenue-maximizing probability of punishment $g$ increases with the share of $L$-type firms and with the amount of disputable transactions $e$. It decreases with the moral cost $c$.

Proof. $\frac{\partial g^*}{\partial \beta} > 0$, $\frac{\partial g^*}{\partial e} > 0$ and $\frac{\partial g^*}{\partial c} < 0$. ■

A revenue-maximizing government will choose a more lenient policy (lower $g$) regarding attempts to exploit tax loopholes when the share of $L$-type firms decreases. Otherwise, a corrupt official would not find it optimal to target $L$-type firms only. A shrinking share of $L$-type firms would induce a corrupt official to make an offer that is attractive to both types of firms (pooling). To avoid this switch in strategy, the government must make it more attractive for $H$-type firms to exploit the tax loopholes without the support of corrupt officials, which can be achieved by increasing the probability of being acquitted ($1-g$). The same mechanism is at work for the social cost $c$. 

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and the amount of disputable transactions \( e \). Typically, the government cannot eliminate all disputable transactions \((e = 0)\) because the tax code cannot take into account all contingencies. The second best strategy to maximize tax revenues is to implement some optimal leniency. Being too tough on potential tax evaders would boost bribes but would not reduce the exploitation of tax loopholes by \( H \)-type firms.

The government may be interested not only in maximizing tax revenues but also in the prevalence of corruption. Suppose for the moment that the government wants to minimize bribe payments and maximize revenues. Then, \( g^* = \beta \cdot \frac{e}{e+c} \) remains the best choice for the government. Increasing the probability for being punished as a tax evader beyond \( g^* \) will induce \( H \)-type firms to choose the bribe option, and total bribes will rise. Figure 3 shows the total bribe as a function of the detection probability \( g \). The government will not benefit in either dimension – tax revenues or fighting corruption – if it closes all tax loopholes.

\emph{Figure 3. Total Bribe Income}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Total Bribe Income}
\end{figure}

Our admittedly simple model suffers from several shortcomings. First, we have focused only on two types of firms. Second, we have made the extreme assumption that all tax officials are corrupt. Third, we have provided the corrupt official with a perfect concealing technique: the corrupt official has the power to avoid any further investigation into the tax matters of those who have paid a bribe. Fourth, we ignore the government’s choice of other factors, such as the remuneration of officials and the punishment of tax evaders. In the following sections, we will discuss some of these extensions.
4. The Prevalence of Corruption

So far, we have made the extreme assumption that all officials are corrupt and willing to assist the taxpayer in concealing illegal or at least disputable accounting transactions. In this section, we discuss the extent to which our finding that some tax loopholes are in the government’s own interest still holds when only a fraction of tax officials are corrupt.

Suppose that only a fraction $\alpha$ of all tax officials are corrupt. All other tax officials report disputable accounting transactions to their superiors, and the cases are settled in court. Firms and tax officials are randomly matched, but firms know whether the official is willing to accept bribes before they submit their tax files.\(^\text{14}\) This extension of our basic model is fairly straightforward. The outcome is simply a linear combination of the outcomes in Sections 2 and 3. Recall that matching between firms and officials occurs before tax files are submitted; hence, firms condition their tax returns on the type of official. A share of $1 - \alpha$ firms will meet honest officials. Hence, all $L$-type firms in this group will avoid all disputable transactions. The $H$-type firms will avoid these transactions if $g > \frac{e}{e+c}$ and accept the risk of being sentenced otherwise. The share $\alpha$ of firms will work with a corrupt official. Hence, the $L$-type firms will always pay the bribe, and the $H$-type firms will join them in the case of pooling, i.e., when $g > \beta \cdot \frac{e}{e+c}$. For lower $g$, there will be separating. The overall revenue loss can be written as follows:

\[
\Delta T = \begin{cases} 
\Delta T_1 & \equiv \alpha \cdot \Delta T_{SEP} + [1 - \alpha] \cdot [1 - \beta] \cdot [1 - g] \cdot e 
& \text{for } g \leq \frac{\beta \cdot e}{e+c} \\
\Delta T_2 & \equiv \alpha \cdot \Delta T_{POOL} + [1 - \alpha] \cdot [1 - \beta] \cdot [1 - g] \cdot e 
& \text{for } \frac{\beta \cdot e}{e+c} < g \leq \frac{e}{e+c} \\
\Delta T_3 & \equiv \alpha \cdot \Delta T_{POOL} 
& \text{for } g > \frac{e}{e+c}
\end{cases}
\]

where $\Delta T_{SEP}$ and $\Delta T_{POOL}$ denote the revenue losses as defined in [4]. In contrast to the basic model, there is now a trade-off for being lenient regarding attempts to exploit tax loopholes ($g < 1$). Leniency may yet be beneficial to curb the tax evasion of $H$-type...

\(^\text{14}\) If job rotation works well, so that firms do not know whether they are confronted with a corrupt official, firms of $L$-type will never submit a tax statement with questionable accounting if the chances of meeting a corrupt official are sufficiently low. In this case, the information asymmetry between firms and official breaks down and the $H$-type firms collude with corrupt officials and evade taxes.
firms that meet corrupt officials. Leniency, however, is also costly because it reduces the revenue from H-type firms that are assigned to honest officials.

**Proposition 3:** With honest and corrupt officials, a government that wants to minimize the expected revenue will not eliminate all tax loopholes. The expected revenue loss for the government is minimized for \( g^* = \beta \cdot \frac{e}{e+c} \) if \( 1 - \alpha < \beta \cdot \frac{e}{e+c} \); \( g^{**} = \frac{e}{e+c} \) otherwise.

Proof. A government that wants to minimize its tax revenue loss will never choose a detection probability \( \beta \cdot \frac{e}{e+c} < g < \frac{e}{e+c} \) because \( \Delta T_{POOL} \geq \Delta T_{SEP} \). Either the government is sufficiently lenient and chooses a small punishment probability \( g^* = \beta \cdot \frac{e}{1+c} \) so that H-type firms do not find it attractive to bribe tax officials, or the government closes most of the tax loopholes \( (g^{**} \geq \frac{e}{e+c}) \) so that all firms engaged with honest officials have no incentive to conduct disputable accounting transactions. This strategy has the disadvantage that all firms monitored by corrupt officials will exploit the tax loopholes \( e \) and pay the bribe. A comparison of \( \Delta T_1(g^* = \beta \cdot \frac{e}{e+c}) \) and \( \Delta T_3 \) yields \( \Delta T_1(g^*) > \Delta T_3 \)
\[
1 - \alpha > \frac{e}{e+c} \quad \blacksquare
\]
If the share of corrupt officials \( \alpha \) is sufficiently high, it will be expensive for the government to be tough on potential tax evaders because the firms will circumvent regulation by bribing tax officials. If most tax officials are honest, however, the government is better off accepting some corruption and closing the loopholes for the majority of taxpayers. The central mechanism from our basic model – tax loopholes may be beneficial for curbing corrupt officials – also holds in this extension of the model.

## 5. Imperfect Concealment

So far, we have assumed that the collusion with a corrupt official perfectly guards a firm from further investigation of its tax files. This assumption was helpful for elaborating the role of tax loopholes in a corrupt environment, but it is clearly unrealistic. Tax officials may have the power to lower the probability of detection and give some tips for better concealing disputable transactions, but the probability of detection can hardly be reduced to zero. In this section, we discuss the implications of imperfect concealing\(^{15}\).

\(^{15}\) Note that \( \Delta T \) is constant for \( g \geq g^{**} \).
technology. Bribing a tax official reduces the probability that a disputable accounting transactions will be detected below unity but does not completely eliminate the risk of being convicted as a tax evader.

As in our basic model, we assume that all tax officials are corrupt \((α = 1)\). If a firm bribes a tax official, the probability of a detection will decrease to \(p\), where \(0 \leq p \leq 1\). Hence, with probability \(p\) the government distrusts the signal of the tax official, and performs a detailed audit. We will refer to \(p\) as the audit probability carried out by the government (or the honest superiors). Without the bribe, the corrupt official forwards the tax files to his or her superiors; hence, the probability of an audit is unity and the tax authority will initiate a lawsuit if \(e > 0\). As described above, the accounting transactions of the \(H\)-type firms will be judged as legal tax avoidance with a probability of \(1 - g\). The \(L\)-type firms will be sentenced for tax evasion with certainty. Hence, with corruption, the net benefit of tax evasion amounts to the following:

\[
\pi_H = e - p \cdot g \cdot [e + c] - b \quad [1']
\]

\[
\pi_L = e - p \cdot [e + c] - b \quad [2']
\]

for the \(H\)- and \(L\)-type firms, respectively. To focus on the interesting cases, we assume \(p < \frac{e}{e+c}\); in other words, the detection probability is sufficiently low that there exists a non-negative bribe where even the \(L\)-type firms find it attractive to exploit the loopholes. As in the previous sections, we first calculate the profit-maximizing bribe for the corrupt official and then determine the government’s revenue-maximizing choice of tax loopholes \((1 - g)\).

**The corrupt official’s profit-maximizing bribe**

A corrupt official’s profit-maximizing bribe depends on the fallback strategy of the \(H\)-type firms as in Section 3. For \(g \leq \frac{e}{e+c}\) the \(H\)-type firms will exploit tax loopholes even without the support of the corrupt officials. For \(g > \frac{e}{e+c}\) the \(H\)-type firms require the support of the corrupt official to exploit tax loopholes. For notational convenience, we define \(\bar{g} \equiv \frac{e}{e+c}\). Now, however, the participation constraint of the \(L\)-type firms matters.

In Section 3, a pooling offer was always attractive for the \(L\)-type firms when it was attractive for the \(H\)-type firms. In other words, the \(L\)-type firms’ willingness to pay for
support from corrupt officials was always higher than that of the H-type firms. This is no longer the case in our extended model. Because there is now a positive probability of detection \( p \), the L-type firms may constrain the corrupt official when setting the bribe. Those firms that, for a given bribe, receive the lower benefits from colluding with the corrupt official determine the pooling offer. Comparing profits \([1']\) and \([2']\) shows that the pooling offer will be determined by the H-type firms \((\pi_H - [e - g \cdot (e + c)] \leq \pi_L)\)\(^{16}\) for

\[
g \leq \frac{e - p \cdot (e + c)}{(1 - p) \cdot (e + c)} \equiv g
\]

and by the L-type firms for \( g > g \). Because \( g < \bar{g} \), we must distinguish three cases when analyzing the corrupt official’s bribe setting. For \( g \leq \bar{g} \) (case 1), the corrupt official can either make a pooling offer that is sufficiently attractive to the H-type firms or target only the L-type firms with a separating offer. For \( \bar{g} < g \leq \bar{g} \) (case 2), the pooling offer must be sufficiently attractive to the L-type firms. Alternatively, the corrupt official can make a separating offer to H-type firms. In cases 1 and 2, the outside option of the H-type firms is to exploit the tax loopholes without the support of the official. For \( g > \bar{g} \) (case 3), the alternatives are the same for the corrupt official as in case 2. However, the outside option of the H-type firms has changed; without the support of the corrupt official, they will not try to exploit the tax loopholes.

**Proposition 4.** Depending on the size of loopholes \((g)\) and the share of L-type firms \((\beta)\), a corrupt official will maximize revenues by adopting the following strategies: (a) For low detection probabilities \((g \leq \bar{g})\), the corrupt official will target the L-type firms if the share of these firms is sufficiently high \((\beta \geq g \cdot \frac{[1-p][e+c]}{e-p[e+c]}\) \). (b) For high detection probabilities, the corrupt official will target the H-type firms if \( \beta \) is sufficiently low \((\beta < 1 - \frac{1}{g} \cdot \frac{e-p[e+c]}{[1-p][e+c]}\) for \( g < \bar{g} \) and \( \beta < \frac{p[1-g][e+c]}{e-p[g][e+c]}\) for \( \bar{g} < g \leq 1\). (c) In all other cases, the corrupt official offers pooling contracts, which attract both type of firms.

Proof. See Appendix A.1.

---

\(^{16}\) H-type firms compare the profits from bribing the official with their outside option, which is avoiding taxes on their own (left hand side). L-type firms gain \( \pi_L \) from bribing the official (right hand side).
Figure 4 illustrates the official’s optimal strategies. For very low detection probabilities ($g \leq \bar{g}$), the corrupt official will target the $L$-type firms if the share of these firms is sufficiently high ($\beta$). Here, it does not pay to make an attractive offer to the $H$-type firms because their outside option – exploiting the tax loopholes without the support of the official – is fairly attractive. The dividing line between separating and pooling is illustrated by the upward sloping line OD in Figure 4 (see condition [A.1]). The corrupt official may also employ a separating strategy, if the detection probabilities are higher ($g < \bar{g} \leq 1$). Here, however, the corrupt official targets the $H$-type firms. This strategy only pays-off for sufficiently high shares of $H$-type firms (low $\beta$; see conditions [A.2] and [A.3] in the Appendix for details). In all other cases, the corrupt official chooses pooling. In Figure 4, this border between pooling and separating is illustrated by the curve HEF. As in our basic model, the corrupt official will find it profitable to target the $L$-type firms when the outside option of the $H$-type firms is attractive. In contrast to the basic model, however, pooling does not prevail in all other cases. The new type of separating contract emerges as the positive detection probability $p$ depresses the willingness to pay of the $L$-type firms.

Figure 4. Pooling, Separating and Optimal Tax Loopholes

The government’s design of tax loopholes

We now turn to the question of whether the government should close all tax loopholes. The government chooses the level $g$ for which the total revenue loss is minimized. We
calculate this revenue loss for each case depicted in Figure 4. Let \( g_1 \equiv \beta \cdot \frac{e^{-p[e+c]}}{[1-p][e+c]} \) denote the border OD between separating the L-type firms and pooling (see condition [A.1]); let \( g_2 \equiv \frac{e^{-p[e+c]}}{[1-\beta][1-p][e+c]} \) denote the left-hand-side border HE between separating of the H-type firms (see condition [A.2]) and pooling and let \( g_3 \equiv \frac{1}{1-\beta} \cdot [1 - \beta \cdot \frac{e}{p[e+c]}] \) denote the right-hand-side border EF (see condition [A.3]). By collecting the terms for tax evasion, we obtain the expected revenue loss for the government:

\[
\Delta T(g) = \begin{cases} 
[1 - \beta \cdot p - [1 - \beta] \cdot g] \cdot e & \text{for } 0 \leq g \leq g_1 \\
[1 - \beta \cdot p - [1 - \beta] \cdot p \cdot g] \cdot e & \text{for } g_1 \leq g \leq g_2 \\
[1 - \beta - [1 - \beta] \cdot p \cdot g] \cdot e & \text{for } g_2 < g \leq g_3 \\
[1 - \beta \cdot p - [1 - \beta] \cdot p \cdot g] \cdot e & \text{for } g_3 < g \leq 1
\end{cases}
\]  

[7]

Note that for sufficiently high levels of \( \beta \), the corrupt official will never find it optimal to target the H-type firms via a separating contract (see Figure 4).

**Proposition 5** With imperfect concealment of tax evasion by corrupt tax officials, a government that wants to minimize the expected revenue loss will: (a) not eliminate all tax loopholes for \( \beta \leq \frac{p \cdot c}{[1-p]e} \). The expected revenue loss for the government is minimized when \( g = g_3 \) which leads to separating of H-type firms. (b) eliminate all tax loopholes (\( g = 1 \)) for \( \frac{p \cdot c}{[1-p]e} < \beta \leq \frac{p[1-p][e+c]}{e-p[e+c]} \); this induces the corrupt official to offer a pooling contract. (c) not eliminate all loopholes for \( \beta \geq \frac{p[1-p][e+c]}{e-p[e+c]} \); the expected revenue loss for the government is minimized when \( g = g_1 \) which leads to separating of L-type firms.

Proof. See Appendix A.2.

The optimal solutions for the government are illustrated by the bold lines in Figure 4. As in the basic model, it is optimal for a revenue-maximizing government to accept large tax loopholes (cf. DJ in Figure 4), when there are many L-type firms. Closing tax loopholes would induce the corrupt officials to lower his bribe so that both types of firms are willing to collude with him or her. Hence, decreasing tax loopholes in such a case will only reduce the expected tax collection from H-type firms and will not affect the expected tax collection from L-type firms. The opposite is the case if there are many H-type firms (low \( \beta \)). Here, it may be optimal to maintain small tax loopholes (cf. EF in Figure 4). The corrupt official targets the H-type firms and the government can fully tax
the $L$-type firms. Again, closing loopholes would induce the corrupt official to switch to pooling, and the government would suffer revenue losses. For intermediate cases, it is optimal to close all tax loopholes (cf. KL in Figure 4). Here, the gains from higher tax collection from $H$-type firms (in the case of a detection) compensate for the revenue losses resulting from the collusion of $H$- and $L$-type firms with the corrupt official.

Overall, our extension, which allows for positive audit probabilities, qualitatively yields the same results as the basic model. It remains the case that the government may have no incentive to punish all the firms that attempt to exploit tax loopholes. Whereas this situation is always optimal in the basic model, it remains optimal in the extended model only when the share of $L$-type firms is sufficiently high or low.

6. **AN ALTERNATIVE AUDITING STRATEGY**

So far, we have assumed that the government has to decide only on the size of tax loopholes to maximize tax revenues. The audit probabilities were exogenously given. At the cost of some additional complexity, one could easily endogenize the audit probability $p$. The audit probability captures the degree to which the government mistrust the signal of tax officials. Suspicious tax files are audited with probability one; all tax files that are not reported as suspicious are inspected in detail with probability $p$ by the honest superiors. Clearly, an increase in the audit probability uncovers more wrong-doings and thus leads to additional tax revenues. However, this comes at a cost as each audit consumes resources, especially because honest auditors usually have to have substantially higher wages (Besley & McLaren, 1993). Hence, the audit probability $p$, which we have used as a parameter in our model, can be seen as the result of the government’s net revenue maximization. Let $C(p)$ with $C', C'' > 0$ denote the cost of tax audits by honest superiors as a function of the audit probability. When a government wants to maximize net revenues it will optimally mistrust the signal of tax officials with probability $p^*$ so that $\frac{\partial \Delta T(p^*)}{\partial p} = C'(p^*)$.\footnote{When all officials are corrupt, the revenue losses can be calculated by inserting the optimal tax loopholes into equation [7].} Trusting the reports of tax officials has the benefit that honest tax officials report only suspicious cases and all clean tax files do not have to be inspected a second time. However, it has the disadvantage that some corrupt officials use the rubberstamping of tax files to help tax evaders. The optimal auditing
probability \( p^* \) will depend on the share of corrupt officials, the distribution of firm types etc. but also on the costs of additional auditing.\(^{18}\)

Could the government be better off by neglecting the bureaucrats’ signals and employ a pure randomization in its tax audits? Australia, Canada, Sweden, the U.S. and the U.K regularly perform such random audits to improve their auditing selection algorithms. Truly random audits on a large scale and as standard procedures, however, are admittedly rare (Slemrod, 2007). We will briefly sketch the government’s trade-off between purely random tax audits and the previous scenario with tax loopholes and small audit probabilities.

In the alternative scenario, the government closes all tax loopholes and randomly inspects firms with a few expensive but honest tax officials. Let \( p_0 \) denote this audit probability. To deter firms from evading taxes, the government must choose an audit probability that fulfills \( e - p_0 \cdot (e + c) \geq 0 \). As audits are costly, the government will choose \( p_0 = \frac{e}{e+c} \). This audit strategy will eliminate all attempts of tax evasion but it comes at the cost of more intensive tax auditing. Instead of inspecting a share of \( p^* \) firms, now the government has to inspect a share of \( p_0 > p^* \) firms.\(^{19}\) The comparison between the two strategies boils down to \( \Delta T^\gamma_{p^*} > C\left(\frac{e}{e+c}\right) - C(p^*) \). The simple extension of our model shows that the availability of purely random tax audits does not make tax loopholes obsolete as a strategic tool. Depending on the costs of tax audits, the government may be better off by accepting the exploitation of tax loopholes by some tax payers rather than randomly auditing a larger number of firms.

7. **Conclusion**

We introduce a simple model for tax loopholes that generate borderline cases between tax evasion and tax avoidance. Tax officials may be able to identify whether a firm has tried to exploit such tax loopholes but be unable to assess tax-planning capabilities. More able firms can successfully contest accusations of tax evasion in a lawsuit. A government that maximizes total tax revenues may have an incentive to not close all tax

\(^{18}\) We do not display the full model here as no fundamental new insights can be gained from it.

\(^{19}\) Note that we have assumed that the marginal audit costs are sufficiently large. Then it is optimal for the government to choose a relatively low audit probability (\( \Delta T^\gamma(p) = C'(p) \Rightarrow p^* < e/(e + c) \)). This is not the cases when employing honest officials is very cheap.
loopholes when there are corrupt officials in the administration. Closing tax loopholes may induce some firms to collaborate with corrupt tax officials rather than exploiting tax loopholes on their own. The resulting tax losses may be larger than the gains from less tax avoidance. Furthermore, corruption increases.

The approach of this paper can answer some questions surrounding the use of tax loopholes by firms in the presence of corrupt tax officials but leaves many other questions open for future research. For instance, the paper completely ignores important aspects of efficiency. There are neither entry nor exit decisions for firms. However, when firms are heterogeneous, corruption in the tax administration may force some firms to exit the market. Furthermore, firms may be forced to distort their input choices to exploit tax loopholes, which generates efficiency costs for the economy, as a result. We also neglect dynamic aspects. A corrupt official may learn over time about the tax planning abilities of a firm if there is repeated interaction in audits. Finally, we also ignore the cost of implementing a particular policy, e.g., closing tax loopholes.

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20 Choi & Thum (2003; 2004) discuss the market entry of firms when facing repeated bribery demands from corrupt officials.
Appendix A.1. Proof of Proposition 4

We start with the case \( g \leq \bar{g} \) (case 1). With separating, the corrupt official charges \( b_{SEP}^1 = e - p \cdot (e + c) \), making the \( L \)-type firms indifferent to accepting or rejecting the offer (\( \pi_L = 0 \)). With pooling, the corrupt official must make the \( H \)-type firms indifferent to accepting the offer \( (e - p \cdot g \cdot (e + c) - b) \) or evading on their own without the support of the official \( (e - g \cdot (e + c)) \). Hence, the highest bribe that is accepted by both types of firms is \( b_{POOL}^1 = g \cdot (1 - p) \cdot (e + c) \). The overall revenues from bribes amount to \( B_{SEP}^1 = \beta \cdot (e - p \cdot (e + c)) \) and \( B_{POOL}^1 = g \cdot (1 - p) \cdot (e + c) \) with the separating and pooling strategy, respectively. The profit-maximizing strategy is found by comparing the revenues:

\[
B_{POOL}^1 > B_{SEP}^1 \iff g \cdot (1 - p) \cdot (e + c) > \beta
\]

We turn now to case 2 \( (g < g \leq \bar{g}) \). Here, the corrupt official may either address all firms (pooling) by making a sufficiently attractive offer to the \( L \)-type firms or focus on the \( H \)-type firms only (separating). With pooling, the highest bribe that is accepted by the \( L \)-type firms amounts to \( b_{POOL}^2 = e - p \cdot (e + c) \), yielding a bribe revenue of the corrupt official of \( B_{POOL}^2 = e - p \cdot (e + c) \). A higher bribe than \( b_{POOL}^2 \) would be accepted only by the \( H \)-type firms. The outside option of the \( H \)-type firms is to exploit the tax loopholes without the support of the official. To maximize his earnings from separating, the corrupt official must make the \( H \)-type firms indifferent to accepting the offer \( (e - p \cdot g \cdot (e + c) - b) \) or evading on their own without the official’s support \( (e - g \cdot (e + c)) \). Therefore, the corrupt official charges \( b_{SEP}^2 = g \cdot (1 - p) \cdot (e + c) \) with separating and earns \( B_{SEP}^2 = (1 - \beta) \cdot g \cdot (1 - p) \cdot (e + c) \). Comparing the official’s revenues leads to the following:

\[
B_{POOL}^2 > B_{SEP}^2 \iff \beta > \frac{1 - \frac{1}{g} \cdot \frac{e - p \cdot (e + c)}{e - p \cdot (e + c)}}{1 - \frac{1}{g} \cdot \frac{e - p \cdot (e + c)}{e - p \cdot (e + c)}}
\]

Note that the right-hand side of [A.2] is concave in \( g \), becomes zero for \( g = \bar{g} \), and reaches \( \frac{pc}{[1-p]e} \) for \( g = \bar{g} \). Because the willingness to pay of the \( L \)-type firms is the...
constraining factor when setting the pooling bribe, pooling pays off only when the share of L-type firms is sufficiently high.

Finally, for $g > \frac{e}{e+c}$ (case 3), none of the firms would carry out disputable accounting transactions without the support of corrupt officials. Again, the corrupt official can target both types of firms or only one type when making an offer. The more attractive partners for the separating strategy are again the H-type firms because their net benefit from tax evasion is higher and the corrupt official can appropriate these net benefits by setting the bribe accordingly. With separating, the bribe amounts to $b^3_{SEP} = e - p \cdot g \cdot [e + c]$ making the H-type indifferent to accepting or rejecting the offer ($\pi_H = 0$). The total revenue for the corrupt official is $B^3_{SEP} = [1 - \beta] \cdot [e - p \cdot g \cdot [e + c]]$. With pooling, the corrupt official charges the highest bribe that is accepted by both types of firms. Hence, the official must make the L-type firms indifferent to accepting or rejecting the offer: $b^3_{POOL} = e - p \cdot [e + c]$. The revenue from bribes becomes $B^3_{POOL} = e - p \cdot [e + c]$. To find the profit-maximizing strategy for the tax official, we compare the bribe revenues:

$$B^3_{POOL} > B^3_{SEP} \iff \beta > \frac{p \cdot [1 - g] \cdot [e + c]}{e - p \cdot g \cdot [e + c]}$$  \hspace{1cm} [A.3]$$

Condition [A.3] describes the borderline cases between pooling and separating. The right-hand side of [A.3] decreases and is concave in $g$; it begins at $\frac{p \cdot c}{[1-p]e}$ for $g = \bar{g}$ and becomes zero for $g = 1$.

## Appendix A.2. Proof of Proposition 5

It is easy to establish that $g_1, g_3$ and $g=1$ are the only candidates to minimize revenue losses. In each range, the revenue losses decline in $g$. Hence, only the upper limits of each range in [7] are candidates for revenue minimization. Because the condition for the revenue loss is the same in both pooling cases, only $g=1$ can be optimal with pooling. (a)

This leaves us with $g_1$ (line 0I in Figure 4), $g_3$ (line EF in Figure 4) and $g=1$ if both types of separating are feasible. Inserting $g_3$ in the third line of [7] yields $\Delta T(g_3) = [1 - p - \beta \cdot \frac{c}{e+c}] \cdot e$. Using $g=1$ in the fourth line leads to $\Delta T(1) = [1 - p] \cdot e$. A comparison immediately yields $\Delta T(g_3) < \Delta T(1)$. Evaluating the first line of [7] at $g_1$ yields $\Delta T(g_1) =$
\[1 - \beta \cdot p - [1 - \beta] \cdot g_1 \cdot e. \] Using \(g=1\) in the fourth line leads to \(\Delta T(1) = [1 - p] \cdot e. \) A comparison yields \(\Delta T(g_1) > \Delta T(1) \iff \beta < \frac{p \cdot [1 - p] \cdot [e + c]}{e - p \cdot [e + c]}. \) To prove that \(\Delta T(g_1) < \Delta T(1), \) we must show \(\frac{p \cdot c}{[1 - p] \cdot e} < \frac{p \cdot [1 - p] \cdot [e + c]}{e - p \cdot [e + c]} \) as \(\beta \leq \frac{p \cdot c}{[1 - p] \cdot e}. \) We compare the numerators and denominators separately. \(\frac{p \cdot c}{[1 - p] \cdot e} < \frac{p \cdot [1 - p] \cdot [e + c]}{e - p \cdot [e + c]} \) is always true if \(p \cdot c < p \cdot (1 - p) \cdot (e + c)\) and \((1 - p) \cdot e > e \cdot p \cdot (e + c). \) The first inequality holds because we have assumed that the detection probability is sufficiently low that the \(L\)-type firms are willing to exploit the tax loopholes with the support of a corrupt official \((p < \frac{e}{e + c}). \) The second inequality holds because \(0 > -p \cdot c. \) Hence, for \(\beta \leq \frac{p \cdot c}{[1 - p] \cdot e}, \) we have \(\Delta T(g_1) > \Delta T(1) > \Delta T(g_3); \) the revenue loss is lowest with separating of \(H\)-type firms. (b and c) Evaluating the first line of [7] at \(g_1\) yields \(\Delta T(g_1) = [1 - \beta \cdot p - [1 - \beta] \cdot g_1] \cdot e. \) Using \(g=1\) in the fourth line leads to \(\Delta T(1) = [1 - p] \cdot e. \) A comparison immediately yields \(\Delta T(g_1) \leq \Delta T(1) \iff \beta \geq \frac{p \cdot [1 - p] \cdot [e + c]}{e - p \cdot [e + c]} \) and vice versa. \(\blacksquare\)

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